**Problem 1**

The eigenstates for a SINGLE neutron are

\[ \psi_n(x) = A_n \sin\left(\frac{n_1 \pi x_1}{L_1}\right), \quad n = 1, 2, \ldots \] with \( E_n = \frac{\hbar^2 n^2}{2mL^2} \)

where \( \psi^+ = \psi(x) \otimes \uparrow \), and \( \psi^- = \psi(x) \otimes \downarrow \).

Since the spin doesn't influence the energy eigenvalues, we can factorize the 2-neutron wave function into a spatial and a spin part: \( \psi(x_1, x_2) | S, M_S > \)

where either \( S = 0, M_S = 0 \) (antisymmetric) or \( S = 1, M_S = -1, 0, \) or \( +1 \) (symmetric). Therefore, \( \psi \) must be symmetric in \( x_1, x_2 \) if \( S = 0 \) and antisymmetric otherwise.

Finally, we can factorize \( \psi(x_1, x_2) \) as

\[ \frac{1}{\sqrt{2}} \left[ \psi_m(x_1) \psi_n(x_2) + \psi_n(x_1) \psi_m(x_2) \right] \text{ (symmetric)} \]

or \[ \frac{1}{\sqrt{2}} \left[ \psi_m(x_1) \psi_n(x_2) - \psi_n(x_1) \psi_m(x_2) \right] \text{ (antisymmetric)} \]

If \( m = n \); otherwise \( \rightarrow \) \( \psi_n(x_1) \psi_n(x_2) \) (symmetric only).

The eigenvalues of the 2-neutron Hamiltonian \( H = H_1 + H_2 \) are the same for the first 2 cases:

- For \( n = m \): \( E_{n,m} = \frac{\hbar^2}{2mL^2} (n^2 + m^2) \)
- For \( n \neq m \): \( E_{n,m} = \frac{2\hbar^2}{2mL^2} n^2 \) otherwise (const case).

Obviously, the lowest possible energy corresponds to \( n = m = 1 \)

\[ \Rightarrow \] 1 non-degenerate ground state: \( \psi_1(x_1) \psi_1(x_2) | S=0, M_S=0 > \)

with eigenvalue (energy) \( \frac{2\hbar^2}{2mL^2} \) \( E_{1,1} \).

The next higher energy is \( E_{1,2} = \frac{\hbar^2}{2mL^2} \) \( \rightarrow \).
1) cont'd

with 4 eigenstates (4x degenerate):

\[ \frac{1}{\sqrt{2}} \left[ \Psi_1(x_1) \Psi_2(x_2) + \Psi_2(x_1) \Psi_1(x_2) \right] \text{LS}=0, m_s=0 > \text{ and } \]

\[ \frac{1}{\sqrt{2}} \left[ \Psi_1(x_1) \Psi_2(x_2) - \Psi_2(x_1) \Psi_1(x_2) \right] \text{LS}=1, m_s=+1 > \]

For the next higher energy, we have

\[ E_{2,2} = \frac{\hbar^2}{2mL^2} (4+4) \]

which is lower than

\[ E_{1,3} = \frac{\hbar^2}{2mL^2} (1+3) \].

There is a single (non-degenerate) state

\[ \Psi_2(x_1) \Psi_2(x_2) \text{LS}=0, m_s=0 > \]

for \( E_{2,2} \) and 4 states for \( E_{1,3} \) (one \( S=0 \), spherically

symmetric, \( J=3 \), \( S=1 \), spherically and axial symmetric) and so on.

(The next eigenvalues in ascending order are \( E_{2,3}, E_{1,4}, E_{3,3}, E_{2,4}, \ldots \))

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**Problem 2**

1) Since the Hamiltonian in region II \((x<0)\) contains the

term \(-\gamma \hat{B} \cdot \hat{S} = -\gamma B S_z\), we must find simultaneous

eigenfunctions of \( H \) and \( S_z \). So for \( m_s=+\frac{1}{2} \),

the Schrödinger equation reads

\[ (x) - \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi_I^+(x) = E \Psi_I^+(x), \quad x<0 \]

\[ (x) - \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi_I^-(x) + \gamma B \frac{\hbar}{2} \Psi_I^+(x) = E \Psi_I^-(x), \quad x\geq0 \]

Let \( k_0 = \frac{p}{\hbar} = \frac{\sqrt{2mE}}{\hbar} \) and \( k_1 = \frac{\sqrt{2m(E+\gamma B^2/2)}}{\hbar} \)

\[ \Rightarrow \Psi_I^+ (x) = e^{ik_0 x} + A_+ e^{-ik_0 x} \quad \text{and} \quad \Psi_I^- (x) = B_+ e^{ik_1 x} \]
2.1  \text{with} \\
\begin{align*}
x &= 0; \quad 1 + A_+ = B_+; \quad k_0(1 - A_+) = k_1 B_+ = k_1 (1 + A_+) \\
\Rightarrow k_0 - k_1 &= (k_0 + k_1) A_+ \quad \text{or} \quad A_+ = -\frac{k_1 - k_0}{k_1 + k_0} \\
B_+ &= \frac{2k_0}{k_1 + k_0}
\end{align*}

2.2: \quad T_+ = \frac{k_1}{k_0} |B_+|^2 = \frac{j \chi(x > 0)}{\omega} = \frac{4k_0k_1}{(k_1 + k_0)^2}

\quad R_+ = |A_+|^2 = \frac{(k_1 - k_0)^2}{(k_1 + k_0)^2} \quad ; \quad R_+ + T_+ = 1

\text{Problem 3}

\begin{align*}
&\text{1} \Rightarrow n (\tau) &= |d_n (\tau)|^2 \quad \text{with} \quad d_n (\tau) = \frac{1}{i \hbar} \int_0^\tau e^{i \omega_0 t'} <n|e^{-i \omega \tau'}|0> \text{d}t' \\
\text{with} \quad \omega_0 &= \frac{E_n - E_0}{\hbar} = n \omega_0 \Rightarrow d_n (\tau) = \frac{e^{i E_0 \tau}}{i \hbar} \int_0^\tau e^{i(n \omega_0 - \omega_0) t'} \text{d}t' <n|1x10> \\
\text{Since} \quad x &= \sqrt{\frac{\tau}{2m \omega_0}} \quad (a^+ a) <n|1x10> = 0 \quad \text{for} \quad n > 1 \\
\text{and} \quad &= \sqrt{\frac{\tau}{2m \omega_0}} \quad \text{for} \quad n = 1. \quad (\text{we also know that} \quad d_n (\tau) \\
\text{becomes large only if} \quad \omega_0 > \omega_p) \Rightarrow n = 1 \quad \text{again.}
\end{align*}

\Rightarrow P_0 \rightarrow n = \frac{e^2 E_0^2}{\hbar^2 2m \omega_0} \left| \int_0^\tau e^{-i(\omega_0 - \omega_p) t'} \text{d}t' \right|^2 = \frac{e^2 E_0^2}{2 \hbar^2 m \omega_0} \left| \frac{e^{i(\omega_0 - \omega_p) \tau} - 1}{i(\omega_0 - \omega_p)} \right|^2 = \frac{e^2 E_0^2 \tau^2}{2 \hbar^2 m} \left( \frac{\sin[(\omega_0 - \omega_p) \tau/2]}{(\omega_0 - \omega_p) \tau/2} \right)^2
Problem 3 cont'd

For small $T$, $P_0 \sim \frac{\sin[(\omega - \omega_p)T]}{(\omega - \omega_p)^{3/2}} \ll 1$

$\Rightarrow P_{01} \ll \frac{2e^2 \varepsilon_0^2 T^2}{2\hbar m \omega_0}$

("short" means that $T(\omega - \omega_p) \ll 1$

For large $T$ and $\omega_0 - \omega_p \neq 0$, $P_{01} \approx \frac{2e^2 \varepsilon_0^2}{\hbar m \omega_0 (\omega - \omega_p)^2} \sin^2(\omega_0 \omega_p T)$

which oscillates between 0 and a maximum of

$\frac{2e^2 \varepsilon_0^2}{\hbar m \omega_0 (\omega - \omega_p)^2}$

as $T$ increases. Of course, if $\omega_0 = \omega_p$, $T$ will never be "long" and the first result obtains for all $T$ (or, rather, until $P$ is no longer $\ll 1$ and 1st order PT breaks down).

Problem 4

a) For $B = 0$, $H_0 = \frac{\hbar^2}{2M} - \frac{e^2}{r}$ with the hydrogen atom

eigenfunctions $|n, \ell, m\rangle$, $0 \leq n \leq \ell$, $-\ell \leq m \leq \ell$.

Ignoring $\frac{e^2}{2mc^2} A^2$, the addition due to $B \neq 0$ is

$H = \frac{e}{2mc} (\hat{P} \cdot \hat{A} + \hat{A} \cdot \hat{P}) = \frac{e}{2mc} \frac{1}{i} \left( \frac{\partial}{\partial \theta} \cos \theta \frac{\partial}{\partial \phi} \right)$

Clearly $\hat{P}$ commutes with $H$ and we get

$H = \frac{e}{2mc} \frac{1}{i} \left( B \frac{\partial}{\partial \phi} \right) = \frac{e}{2mc} B \hat{L}_z$. 
Problem 4 continued

b) i) \[ \langle 2, 0, 0 | L_z | 2, 0 \rangle = \langle 2, 1, 0 | L_z | 2, 1 \rangle = 0 \]

\[ \implies \text{there is no first-order shift due to } H_p \text{ for these EFs.} \]

ii) \[ \Delta E_{1\text{st order}} = \frac{eB}{2mc} \langle 2, 0, \pm 1 | L_z | 2, 1 \pm 1 \rangle = \pm \frac{eB \hbar}{2mc} \]

iii) The four \( n = 2 \) EFs above are all degenerate in energy (they form a degenerate subspace). To avoid problems, we need a basis of that subspace which diagonalizes \( H_p \).

Fortunately, since the standard hydrogen atom \( \psi \)'s are already eigenstates to \( L_z \) (and thus \( H_p \)), \( H_p \) is already diagonal \( \implies \) nothing to do.

iv) \[ |\psi_{1}\rangle = \sum_{\ell, \mu \nu} \frac{\langle \ell \mu \nu | H_p | 2, \ell, \ell \rangle}{E_{\ell \mu \nu} - E_2} |\ell \mu \nu \rangle \]

However, because all \( |\ell \mu \nu \rangle \) are EF's to \( H_p \) \( (\text{by } 2) \),

this is always zero \( \implies \) there is no \( 1\)st order change in the EFs.

v) For exactly the same reason, there is no \( 2\)nd order change in \( E \) - in fact,

\[ E = -\frac{\mu B}{4} + \frac{eB \hbar}{2mc} \quad \text{to all orders for all } |\ell \mu \nu \rangle \]
Problem 5

a) For the numerical values given, \( \omega_c = \frac{1}{6.58 \times 10^{-16}} \text{ Hz} = 1.52 \times 10^{15} \text{ Hz} \)

This is different from \( \omega_p = 10^{-15} \text{ Hz} \). Because of the \( \delta \)-function in \( \mathcal{F} \mathcal{G} \), \( \frac{dP(i \rightarrow f)}{dt} = 0 \)

b) E.g., a state for two spin \(-\frac{1}{2}\) particles coupled to total spin \( S = 0 \):
\[
\frac{1}{\sqrt{2}} \left( |\frac{1}{2}, \frac{1}{2}\rangle \otimes |\frac{1}{2}, -\frac{1}{2}\rangle - |\frac{1}{2}, -\frac{1}{2}\rangle \otimes |\frac{1}{2}, \frac{1}{2}\rangle \right)
\]

cannot be written as \( (a) \otimes (c) \) = \( \frac{1}{\sqrt{2}} \left[ (0|0\rangle + 0\langle 1|1\rangle \right] \)

for any two \( 1 \)-particle states \((a), (c)\)

c) Since the dipole moment operator \( \mathbf{e} \cdot \mathbf{r} \) is a rank-1 spherical tensor, the Wigner-Eckardt theorem says

\( j \) must be able to couple with \( j \) to yield \( j \) again \( \Rightarrow |j-1| \leq j \leq j+1 \). This is impossible for \( j = 0 \) \((1 \neq 0)\)

but possible for \( j = \frac{1}{2} \) \((\frac{1}{2} = \frac{1}{2}) \Rightarrow j = \frac{1}{2} \) \( \leq \) the maximum.

d) According to the WKB method, \( \psi(x) = \frac{A}{\sqrt{p(x)}} \cdot e^{i\phi(x)} \)

Where \( \phi(x) = \frac{1}{h} \int_{x_0}^{x} (p(x')) dx' \) is some phase.

\( dp(x_1 \ldots) = \frac{|A|^2}{p(x_1)} dx_1 \) and \( dp(x_2 \ldots) = \frac{|A|^2}{p(x_2)} dx_2 \)

\( \Rightarrow \) Ratio = \( \frac{dp(x_2)}{dp(x_1)} = \frac{\sqrt{2m \left( E - V(x_2) \right)}}{\sqrt{2m \left( E - V(x_1) \right)}} \). This also makes sense classically, since \( dp \propto \frac{dx}{\sqrt{V}} \) (time spent near \( x_1 \)).

e) According to the variational method, \( E_0 \leq \min \langle i | H | i \rangle \)

and the basic state \( |ij\rangle \) with the lowest \( \langle i | H | i \rangle \) has the best chance to be "close to" the ground state.
5f: \[ \vec{P} = \frac{1}{4} \hat{x} + \frac{1}{4} \hat{y} + \frac{1}{4} \hat{z} \]

\[ \vec{Q} = \frac{1}{2} \left( \vec{\sigma} \cdot \vec{P} \right) + \frac{1}{2} \sigma_0 = \begin{pmatrix} \frac{5}{8} \\ \frac{1}{8} \end{pmatrix} \begin{pmatrix} \frac{1}{2} \left(1+i \right) \\ 3/8 \end{pmatrix} \]

5g: \[ \frac{d\sigma}{d\Omega} = \frac{\sin^2 \theta_0}{K^2} = \frac{0.00937}{(1 \text{ nm}^{-1})^2} = 10^{-2} \text{ nm}^2 \]

\[ = 10^{-16} \text{ cm}^2 \text{ per sr} \]